

Homework Assignment 6  
Optimal Control- MAE 274  
Prof. Solmaz S. Kia

**Turn in your HW electronically to the respective folder in Canvas. Return your Matlab codes as well.**

**Assignment 1.** Do the following problems from reference [1] (Kirk, see the class syllabus for the list of references)

**5.1 (see below) and 5.14**

**For 5.1 complete also the following assignment** (assume the velocity of the boat is given and constant, and the current velocity is also constant and uniform across the river.) After solving the problem as stated in the problem 5.1, substitute the following numerical values to plot your optimal solution (plot  $x$  vs.  $y$ , and also control history).

Point 1:  $x(0), y(0) = 0$ ,

Point 2:  $x(t_f) = 15m, y(t_f) = 9m$ ,

$V = 3 \frac{m}{s}, s(x) = 0.5 \frac{m}{s}$ ,

Use the Matlab code that is provided for you in the Canvas to solve this problem using Matlab 'bvp4c'. Does the solution match your analytical solution? Plot the results.

**Assignment 2 (Bryson). Maximum Radius Orbit Transfer in Given Time**

Given a rocket engine with constant thrust  $T$ , operating for a given length of time  $t_f$ , we wish to find the thrust-direction history,  $\theta(t)$  to transfer a space craft (S/C) from a given initial circular orbit to the largest possible circular orbit. The nomenclature is defined in the figure below;

$r$ =radial distance of S/C from attraction center,

$u$ = radial component of velocity,  $v$ =tangential component of velocity,

$m$ = mass of S/C.

$-\dot{m}$ =fuel consumption rate (constant),  $\theta$ =

thrust direction angle,

$\mu$ = gravitational constant of attracting center. Using time in

units of  $\sqrt{r_0^3/\mu}$ ,  $r$  in units of  $r_0$ ,  $u$  and  $v$  in

units of  $\sqrt{\mu/r_0}$ ,  $m$  in  $m_0$ , thrust in units of  $\mu m_0/r_0^2$ , the problem may be stated as : find

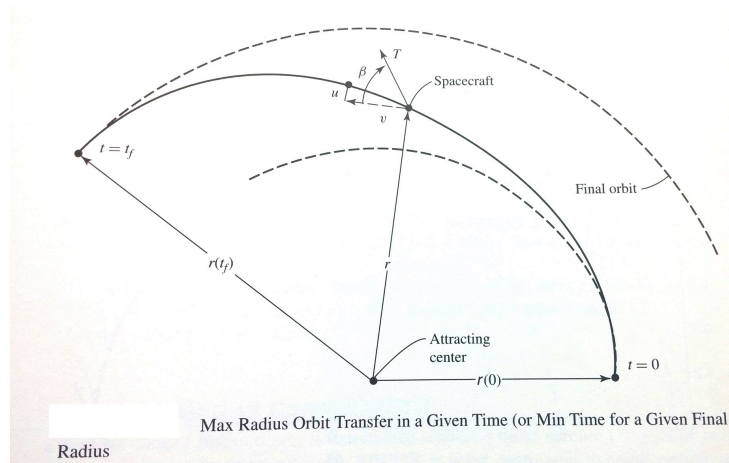
$\theta(t)$  to maximize  $r(t_f)$  (minimize  $-r(t_f)$  ) subject to

$$\dot{r} = u, \quad \dot{u} = \frac{v^2}{r} - \frac{1}{r^2} + a \sin \theta, \quad \dot{v} = -\frac{uv}{r} + a \cos \theta \quad \text{where} \quad a \triangleq \frac{T}{1-|\dot{m}|t}$$

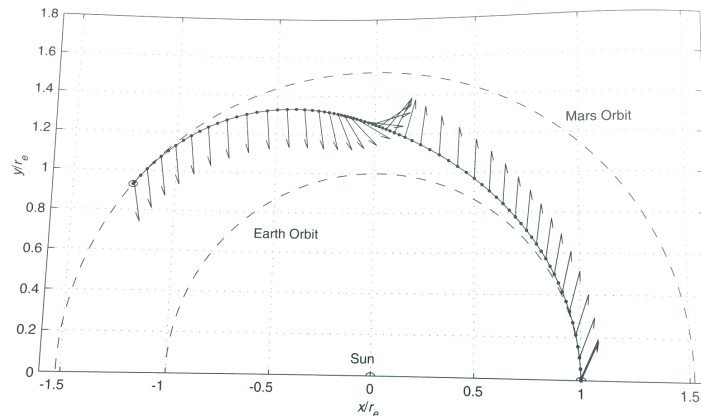
with (recall that  $t_f$  is given and fixed)

$$r(0) = 1, u(0) = 0, v(0) = 1,$$

$$\mathfrak{M}_1 = u(t_f) = 0, \mathfrak{M}_2 = v(t_f) - \frac{1}{\sqrt{r(t_f)}} = 0. \quad (\text{terminal states are constrained (recall } m(x(t_f), t_f)=0 \text{ kind of final constraints)})$$



- Write down the first order necessary conditions of the optimal solution.
- Use Matlab 'bvp4c' to solve this problem for  
 $T = 0.1405, |\dot{m}| = 0.07489, t_f = 3.3155$   
and plot the spacecraft's trajectory.
- (Optional)** Produce a plot similar to the figure below using your optimal control.



Min Time Path from Earth Orbit to Mars Orbit Using Low Thrust; Trip Time = 193 Days

### Assignment 3.

- Design a steady state LQR controller for the system below using  $Q = I_2, R = 0.01$ . Write down the equations for optimal controller, then you can solve the problem using Matlab lqr command.

$$\dot{x} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u.$$

- Let  $y = (1 \ 0)x$ . A naïve way to implement a reference tracker (that is  $y$  tracks a reference signal  $r$ ) for this system is to modify the LQR controller from  $u = Fx$  to  $u = Fx + r$ , which give the closed-loop system

$$\dot{x} = (A + BF)x + Br$$

Through simulation using the gain you obtained in part one, verify that this leads to particularly poor tracking of a unit step input by  $y$ .

- Study the section on Linear Tracking Problem in Ref[1], starting at page 219. Use equations (5.2.40), (5.2.44), (5.2.45), (5.2.47) and (5.2.48) to design a controller for this system which makes  $y$  to track a unit step input over finite horizon  $[0,50]$  seconds. Here you need to pick your own cost function (see example 5.2.3 in ref[1]). Demonstrate trajectories of  $y(t)$  over  $[0,50]$ .
- Investigate if for the parameters you chose in previous part, (5.2.44) and (5.2.44) has steady state solutions. If solutions exists, use the steady state gains as your control gain for the entire time horizon  $[0,50]$  seconds to simulate trajectories of  $y(t)$  over this time interval. How does your plot compares to the plot from previous part?

**Assignment 4 (Aircraft Control and Simulation, F. Lewis).** A linearized model of the short-period dynamics of an advanced fighter aircraft is given as  $\dot{x} = Ax + Bu$ , where

$$A = \begin{bmatrix} -1.341 & 0.9933 & 0 & -0.1689 & -0.2518 \\ 43.223 & -0.8693 & 0 & -17.251 & -1.5766 \\ 1.341 & 0.0067 & 0 & 0.1689 & 0.2518 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}$$

where state  $x = [\alpha \ q \ \gamma \ \delta_e \ \delta_f]^T$  consists of respectively angle of attack, pitch rate, flight path angle, elevator deflection and flaperon deflection. The control inputs are elevator and flaperon commands so that  $u = \begin{bmatrix} u_e \\ u_f \end{bmatrix}$ .

The measured output available for control purposes is  $y = [q \ n_{zp} \ \gamma \ \delta_e \ \delta_f]^T$  where  $n_{zp}$  is normal acceleration at the pilots station. The relation between  $y(t)$  and  $x(t)$  is given by

$$y = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 47.76 & -0.268 & 0 & -4.56 & 4.45 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} x = Cx$$

- Design a infinite horizon LQR control to stabilize this system. Your control should have the form of  $u = Fy$  (what is  $F$  in your design). Choose your weighting factors  $Q$  and  $R$ .
- Simulate the performance of the controller (plot control input and also  $\alpha \ q \ \gamma$  in degrees) when the initial condition of 0.2 rad in angle of attack is consider (the rest of states are zero at initial time) for  $t \in [0,10]$  seconds. Is your choice of  $R$  keeps your control surface deflection commands in  $\pm 15$  degree? If not, can you adjust your choice to meet these bounds?
- Investigate the effect of changing the weights on your control by multiplying your  $R$  with  $\rho = 0.1, 0.5, 10$ . Obtain the LQR gain for each weight selection. Simulate the scenario described in part (b). Overlay your plots for different values of  $R$  so you can see the effect of adjusting the gains.