Homework Assignment 1
Optimal Control- MAE 274
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Turn in your code for problem 4 electronically to Canvas.

Problem 1 (Kirk 2.6). Figure below shows a rocket that is to be approximated by a particle of instantaneous mass m(t).

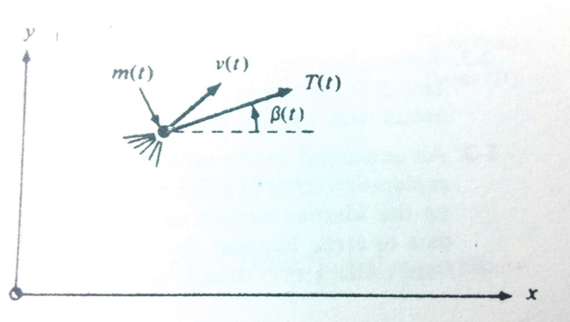

Figure 2-P6

The instantaneous velocity is $v(t), T(t)$ is the thrust, and $\beta(t)$ is the thrust angle. If we assume no aerodynamics or gravitational forces, and if we select $x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}, x_5 = m, u_1 = T, u_2 = \beta$, the state equations are

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2 = \frac{u_1(t)\cos\big(u_2(\mathrm{t})\big)}{x_5(t)}$$
$$\dot{x}_3(t) = x_4(t)$$
$$\dot{x}_4 = \frac{u_1(t)\sin\big(u_2(\mathrm{t})\big)}{x_5(t)}$$
$$\dot{x}_5 = -\frac{1}{c}u_1(t),$$

where c is a constant of proportionality. The rocket starts from rest at the point $x = 0, y = 0$.

a) Determine a set of physically reasonable state and control constraints.
b) Suggest a performance measure, and any additional constraints imposed, if the objective is to make $y(t_f) = 3$ miles and maximize $x(t_f)$; t_f is specified.
c) Suggest a performance measure, and any additional constraints imposed, if it is desired to reach the point $x = 500\ mi, y = 3\ mi$ in 2.5 min with maximum possible vehicle mass.

Problem 2 (review material). Give two criteria to determine if a symmetric real matrix is positive definite or positive semidefinite. Using one of them to test if the following matrix is positive definite

$$A = \begin{bmatrix} 4 & -1 & -2 \\ -1 & 2 & -2 \\ -2 & -2 & 4 \end{bmatrix}.$$

Problem 3.

a) Using the first-order necessary conditions, find a minimum point of the function $F(x, y, z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z + 9$

b) Verify that the point is a local minimum point by verifying that the second-order sufficiency conditions hold.

c) Prove that the point is a global minimum point.

Problem 4. This problem explores the use of steepest descent algorithm and Matlab fminunc function to solve the optimization problem of Problem 3 numerically. Lets re-state the problem as

$$F(x) = F(x_1, x_2, x_3) = 2x_1^2 + x_1x_2 + x_2^2 + x_2x_3 + x_3^2 - 6x_1 - 7x_2 - 8x_3 + 9$$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

The steepest decent algorithm is

$$x(k+1) = x(k) - \alpha(k)F_x\big(x_1(k), x_2(k), x_3(k)\big),$$

where F_x is the gradient of the cost function F.

The underlying idea to pick $\alpha(k)$ is as follows: when you are at point $x(k)$ and you are moving in the direction $-\alpha(k)F_x\big(x_1(k), x_2(k), x_3(k)\big)$ to get to the point $x(k+1)$ you what to pick $\alpha(k)$ such that $G(\alpha(k)) = F\Big(x(k) - \alpha(k)F_x\big(x_1(k), x_2(k), x_3(k)\big)\Big)$ is minimized. There are different methods to obtain this minimum value; you can consult any classic optimization book for more details. In this problem, $G(\alpha(k))$ will be a quadratic function and finding its minimum is pretty straightforward.

a) Using $x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, write out the first iteration of the steepest descent algorithm and obtain the optimum value for $\alpha(0)$. What is the value of the $x(1)$ you get?

b) Write a MATLAB code to solve this problem using a steepest descent algorithm and an initial value of $x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ using optimal $\alpha(k)$ at each time step as well as constant steps $\alpha = 0.1$, $\alpha = 0.5$ $\alpha = 1$. Use $||x(k+1) - x(k)|| \leq 10^{-6}$ as stopping criterion. How many iterations does it take to converge? Explain your results.

c) Use MATLAB's fminunc function to solve for the minimum and compare its performance with your algorithm.