Consistent loosely coupled decentralized cooperative navigation for team of mobile agents

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BIOGRAPHY

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ABSTRACT

In this paper, a cooperative navigation algorithm for team of mobile agents is presented. The proposed algorithm is a loosely coupled decentralized algorithm. The setting we consider consists

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of a set of mobile agents with sensing, processing and communication capabilities. In this setting each agent maintains a local filter to propagate and update its local pose estimates using self-motion measurements and occasional external absolute measurement signals. Whenever, an agent takes a relative measurement with respect to a teammate, our proposed algorithm enables these two agents to jointly process this relative measurements to update their local estimates in a consistent manner in the absence of exact knowledge about the correlation between their local estimates. Simulations demonstrate the benefits of our proposed algorithm.

I. INTRODUCTION

Determining the location of mobile agents (e.g, automobile, aerial vehicle, underwater vehicle, ground robots) has always been a crucial factor for their autonomy in different circumstances, either indoor or outdoor environments. Pose (position and orientation) information is essential in other tasks such as path planning and environment monitoring. Navigation problem for mobile agents consists of estimating the states of the agent using the sensor on board of the agent. The basic idea here is to combine measurements from proprioceptive sensors that monitor the motion of the vehicle with measurements from exteroceptive sensors that observe the surrounding environment. The common navigation techniques rely on Global Positioning System (GPS) [1] or use of external landmarks such as pre-installed beacons or fixed and stationary features in the environment. In beacon-based localization [2] measurements taken from pre-installed markers with known locations are used to improve the self-localization of mobile agents. The simultaneous localization and map building (SLAM) techniques [3] use vision or LIDAR based measurements from distinguishable features (landmarks) in the environment to build a map of the environment and localize the agents in that map. These navigation techniques because of their reliance on external services and features that may not necessarily be available in every environment do not fully address the mobile agent localization problem.

An alternative navigation technique is cooperative navigation or cooperative localization (as it is more often referred too) [4]–[14]). In cooperative navigation, mobile agents, with communication and computation capabilities, jointly process a relative measurement between each other (no dependency on external features and services outside the team) to increase their localization accuracy. Despite the great appeal of cooperative navigation for localization of team of networked mobile agents in GPS and land-marked challenged environments, its integration in real world applications has been challenging. Due to relative measurements updates, cooperative localization results in strong couplings/cross correlations between the team members' pose estimates. Accounting for these coupling/cross correlations is crucial for filter consistency. It is well known that disregarding the past correlations causes the so-called rumor propagation phenomenon that can lead to overconfidence and, even to divergence of the estimates, as reported in [15]. However, exact account of cross correlation terms comes with high computation and communication cost, stringent requirement on network connectivity and communication channel utilization. In an effort to develop practical cooperative localization algorithms, in this paper we propose a loosely coupled algorithm where the correlations are accounted for in an implicit manner by conservative but consistent estimate of joint covariance matrix of team members.

Related work: Under the assumption of Gaussian distribution for process and measurement noises, [6] and [16] proposed a centralized Extended Kalman Filter (EKF) based cooperative localization to propagate and update the estimates of all the agents in a jointly manner. Other centralized cooperative localization algorithms to deal with non-Gaussian noises are discussed in [12] and [13]. In a centralized cooperative localization algorithm, a fusion center (FC) which is either a leader agent or a center overseeing the operation, is required. At each time step, FC collects the individual motion measurements and agent-to-agent relative measurements to estimate the team members poses or generate the update commands for the team members. Then, the FC sends back this information to each agent In centralized operations the computation and communication costs scale poorly with respect to the number of the agents in the team.

In recent years, to avoid single point failure and energy inefficiency of central operations, many effort have been devoted to develop decentralized cooperative navigation algorithms. As discussed in [14], the approaches to design decentralized cooperative localization algorithms can be divided into two categories: tightly coupled algorithms and loosely coupled algorithms. In tightly coupled approaches, normally the computations of a central cooperative localization is distributed among the team members. Some examples of tightly coupled cooperative localization algorithms can be found in ([17], [18], [19], [13], [20]), where EKF based , Uncented Kalman filter baed and maximum a posteriori based cooperative localization is decentralized among team members. Tightly coupled algorithms, because of an accurate account of correlations provide higher positioning accuracy but come with higher communication cost and stringent network connectivity conditions. To avoid these costs, loosely coupled cooperative localization methodologies, where the correlation terms are not maintained but accounted for in an implicit way are proposed in the literature. In [21], an interleaved update algorithm was proposed to provide consistent estimate that only the agent obtaining the relative measurement updates its state. A bank of EKFs is maintained at each agent. Using an accurate bookkeeping of the identity of the agents involved in previous updates and the age of such information, each of these filters is only updated when its propagated state is not correlated to the state involved in the current update equation. The main drawback is the growing size of information needed at each update time which increases the computational complexity of the algorithm. Other loosely couple cooperative localization algorithms mostly use covariance intersection method (c.f. [22], [23]) in their approaches. Recall that the covariance intersection method fuses two or more tracks from same process when the correlations between tracks are unknown. However, in cooperative localization, the local pose estimates of two different mobile agents are jointly updated based on the feedback from a relative measurement between them. As a result, cooperative localization techniques which use covariance intersection method assume that each agent keeps a copy of the state estimate of the entire team locally. See for example [15] which uses such an approach for the localization of a group of space vehicles communicating over a fixed ring topology. Another example of the use of split covariance intersection is given in [24] for intelligent transportation vehicles localization. [25] uses a common past-invariant ensemble Kalman pose estimation filter in another loosely-coupled CL approach for intelligent vehicles. This algorithm is very similar to the decentralized covariance intersection data fusion method described above, with the main difference that it operates with ensembles instead of with means and covariances. In a different approach, to avoid the requirement for each agent to keep a copy of state estimate of the entire team, [26] proposes an algorithm in which an agent taking relative pose measurement uses this measurement and its current pose estimate to obtain and broadcast a pose and the associated error covariance of its landmark agent (the landmark agent is the agent the relative measurement is taken from). Then, the landmark agent uses the covariance intersection method to fuse the newly acquired pose estimate with its own current estimate to increase its estimation accuracy. This technique crucially relies on relative pose measurements and cannot be applied for the more common cases of relative range and relative bearing measurements.

Contribution of the paper: In this paper, we develop a cooperative navigation method in which correlations are not maintained explicitly but are accounted for in an implicit manner using proper conservative but consistent estimate of the joint covariance of the agents. In our proposed algorithm each agent maintains its local pose estimation algorithm which is propagated using self motion-measurements and updated locally if occasional absolute measurements become available, e.g., through occasional access to GPS signals. Whenever, an agent takes a relative measurement from another agent in the team, this relative measurement is processed cooperatively by the two agents involved using their current propagated state and covariance matrices. Our approach does not require centralized data storage and processing. Moreover, it does not enforce a particular communication hierarchy or topology and individual agents can join and leave the group without the need to be aware of previous communications or the size of the group. Because, each agent maintains its own local filter, our method is robust to communication failure. Finally, unlike some other loosely coupled decentralized algorithms that only work for specific types of relative measurement, our algorithm can process any form of relative measurement, i.e., there is no restriction on the type of relative measurement for our algorithm. In short our main contribution is a cooperative localization method that can be used as an add-on augmentation to improve self-localization of the mobile agents. That is, agents can implement any localization strategy such as dead-reckoning, GPS or SLAM and when the accuracy via these methods is not satisfactory, they can seek assistance from other agents in their communication and relative measurement device range without compromising the estimation consistency.

The outline of the paper is as follows: Section II describes our notations, terminologies and a crucial lemma that we use to develop our results. Section III our loosely coupled decentralized cooperative navigation algorithm's derivation and its rigorous consistency analysis. We also discuss two possible scenarios to implement our proposed algorithm. Section IV presents our simulation studies.

II. **PRELIMINARIES**

This section described our notations, terminologies and a preliminary lemma we use in our development in the proceeding sections.

Notations: the set of real and non-negative integer numbers are, respectively, $\mathbb{R}$ and $\mathbb{Z}^{++}$. The set of $n\times n$ real positive definite matrices is $\mathbb{S}_n^{++}$. The transpose of matrix $\mathbf{A}\in\mathbb{R}^{n\times m}$ is $\mathbf{A}^\top$. For a matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, its trace is $\mathrm{Tr}(\mathbf{A})$ and its determinant is $\det(\mathbf{A})$. Considering a team of N mobile agents, the local variables of agent $i \in \{1, \cdots, N\}$ are distinguished by the superscript i, e.g., $\mathbf{x}^i$ is the state (e.g., position and orientation) of agent i, $\hat{\mathbf{x}}^i$ is its state estimate, and $\mathbf{P}^i$ is the covariance matrix of its state estimate, where $\mathbf{x}^i$, $\hat{\mathbf{x}}^i \in \mathbb{R}^{n^i}$ and $\mathbf{P}^i \in \mathbb{S}^{++}_{n^i}$. We use the term *cross-covariance* to refer to the correlation terms between two agents in the joint covariance matrix between them. We demonstrate the cross-covariance of the state vectors of agents i and j by $\mathbf{P}_{ij} \in \mathbb{R}^{n^i\times n^j}$. We denote the propagated and updated variables, say $\hat{\mathbf{x}}^i$, at time-step k by $\hat{\mathbf{x}}^{i\text{-}}(k)$ and $\hat{\mathbf{x}}^{i+}(k)$, respectively. We drop the time-step argument of the variables as well as matrix dimensions whenever they are clear from the context. Next, we provide our definition of estimator consistency which conforms with the definition given in [27].

Definition 1 (Consistency of an estimate): Given a process with state $\mathbf{x}$, an state estimator which produces estimate $\hat{\mathbf{x}}$ with associated error covariance $\mathbf{P}$ is said to be consistent if it is unbiased, i.e., $\mathbf{E}[\mathbf{x}-\hat{\mathbf{x}}]=0$ and its state estimation error satisfies $\mathbf{P} \geq \mathbf{E}[(\mathbf{x}-\hat{\mathbf{x}})(\mathbf{x}-\hat{\mathbf{x}})^\top]$.

We should mention here that in some references the definition of estimator consistency includes covariance matching condition $\mathbf{P} = \mathbf{E}[(\mathbf{x}-\hat{\mathbf{x}})(\mathbf{x}-\hat{\mathbf{x}})^\top]$ (see e.g., [28], [29]).

In our developments below we use the following lemma.

Lemma 2.1 (A block diagonal upper bound on positive semi-definite matrices [30, page 207 and page 350]): Consider $\mathbf{P}_1 \in \mathbb{S}^{++}_{n_1}$, $\mathbf{P}_2 \in \mathbb{S}^{++}_{n_2}$ and $\mathbf{X} \in \mathbb{R}^{n_1\times n_2}$ such that

$$\begin{bmatrix} \mathbf{P}_1 & \mathbf{X} \\ \mathbf{X}^\top & \mathbf{P}_2 \end{bmatrix} \geq 0.$$

Then, for any $\omega \in [0,1]$, we have

$$\begin{bmatrix} \frac{1}{\omega}\mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \frac{1}{1-\omega}\mathbf{P}_2 \end{bmatrix} \geq \begin{bmatrix} \mathbf{P}_1 & \mathbf{X} \\ \mathbf{X}^\top & \mathbf{P}_2 \end{bmatrix}.$$

III. A LOOSELY COUPLED CONSISTENT COOPERATIVE LOCALIZATION ALGORITHM

In this section we present our decentralized cooperative localization algorithm for a team of mobile agents.

A. Description of the mobile agent team and problem setup

Consider a team of N collaborating mobile agents moving in a M-dimensional space. In this team, each agent is capable of processing, measuring, and communicating. Each agent is equipped with proprioceptive sensors (e.g., wheel encoders) that provide measurement of ego-motion, and exteroceptive sensors (e.g., laser and vision) which enables the mobile agents to take relative measurements from other team members. Also, each agent has a detectable unique identification (UID), which here without loss of generality we assume to an integer $i \in \{1, \cdots, N\}$. The motion of each agent is modeled as a discrete-time time-varying linear system described by

$$\mathbf{x}^i(k+1) = \mathbf{F}^i(k)\mathbf{x}^i(k) + \mathbf{B}^i(k)\mathbf{u}^i(k) + \mathbf{G}^i(k)\boldsymbol{\eta}^i(k), \quad k \in \mathbb{Z}^{++}, \tag{1}$$

where $\mathbf{x}^i \in \mathbb{R}^{n^i}$, $\mathbf{u}^i \in \mathbb{R}^{m^i}$, and $\boldsymbol{\eta}^i \in \mathbb{R}^{p^i}$ are, respectively, the state, the control input and the process noise of agent i. Here the process noise, $\boldsymbol{\eta}^i$, $i \in \{1, \cdots, N\}$, is an independent zero-mean white Gaussian process with a known positive definite diagonal variance $\mathbf{Q}^i(k) = E[\boldsymbol{\eta}^i(k)\,\boldsymbol{\eta}^i(k)^\top]$ and uncorrelated in time. There is no requirement on the robotic team to be homogeneous. $\mathbf{F}^i \in \mathbb{R}^{n^i \times n^i}$, $\mathbf{B}^i \in \mathbb{R}^{n^i \times m^i}$ are the system matrices while $\mathbf{G}^i \in \mathbb{R}^{n^i \times p^i}$ is the coefficient matrix of process noise. It is also assumed that the process noise of each agent pair i and j are mutually independent.

Every agent also carries exteroceptive sensors to detect, uniquely, the other agents in the team and take relative measurements from them when they come to its limited measurement range. The relative measurement collected by robot i from robot j is described by

$$\mathbf{z}_{ij}(k) = \mathbf{H}^i_{ij}(k)\mathbf{x}^i(k) + \mathbf{H}^j_{ij}(k)\mathbf{x}^j(k) + \boldsymbol{\nu}^i_{\mathrm{r}}(k), \quad \mathbf{z}_{ij} \in \mathbb{R}^{n^i_{\mathrm{rz}}}, \tag{2}$$

Every agent $i \in \{1, \cdots, N\}$ also has sensors on-board to collect absolute state measurement when the opportunity to collect such measurement is present. We assume that such occasions are limited. The absolute measurement model is as

$$\mathbf{z}_i(k) = \mathbf{H}_i(k)\mathbf{x}^i(k) + \boldsymbol{\nu}^i_{\mathrm{a}}(k), \quad \mathbf{z}_i \in \mathbb{R}^{n^i_{\mathrm{az}}}, \tag{3}$$

$\mathbf{H}^i_{ij}$, $\mathbf{H}^j_{ij}$ and $\mathbf{H}_i$ are the measurement matrices of, respectively, relative and absolute measurement. $\boldsymbol{\nu}^i_{\mathrm{r}}$ and $\boldsymbol{\nu}^i_{\mathrm{a}}$ are the respectively relative and absolute measurement noises of robot $i \in \{1, \cdots, N\}$ that are both zero-mean white Gaussian processes with known diagonal covariance matrices $\mathbf{R}^i_{\mathrm{r}}(k)$ and $\mathbf{R}^i_{\mathrm{a}}(k)$. To be mentioned, all noises are assumed to be white and mutually uncorrelated. Here, we assume that all the sensor measurements are synchronized.

For simplicity, in our aforementioned system model we used linear system model to describe the mobile agents dynamics and measurement model. However, one may linearize a nonlinear dynamics model about previous estimates to obtain the system matrices $\mathbf{F}^i$, $\mathbf{B}^i$, $\mathbf{G}^i$, $\mathbf{H}^i_{ij}$ etc, for all $k \in \mathbb{Z}^{++}$, see e.g. [19].

B. **Consistent cooperative navigation under Unknown correlation**

In a network of mobile agents deployed in a harsh environment, network connectivity is dynamic. Communication failure due to obstacles and also agent's limited communication is always present. Additionally, mobile agents may join or leave the network, or may become unavailable because of unpredictable failures or obstructions in the environment. All these considerations make the design of effective tightly coupled cooperative localization algorithms challenging. To address concerns about network connectivity, in the following, we propose a loosely coupled cooperative localization algorithm in which we do not impose any connectivity condition for the team. The setting in our algorithm is outline below: each agent $i \in \{1, \cdots, N\}$ maintains its own local estimator to compute its pose estimate and the corresponding error covariance without the need to communicate with other agents. When at any time k agent i takes a relative measurement with respect to another agent j, it sends its local estimates and measurement information to agent j. Agent j uses this information along its own local estimate to generate a consistent update for both of the agents involved in the relative measurement. The updated pose estimate for agent i then is sent back to that. In our setting, these two agents in the team only need to communicate with each other when they want to process a relative measurement between themselves (see Fig. 1). Normally communication range of mobile agents is larger than

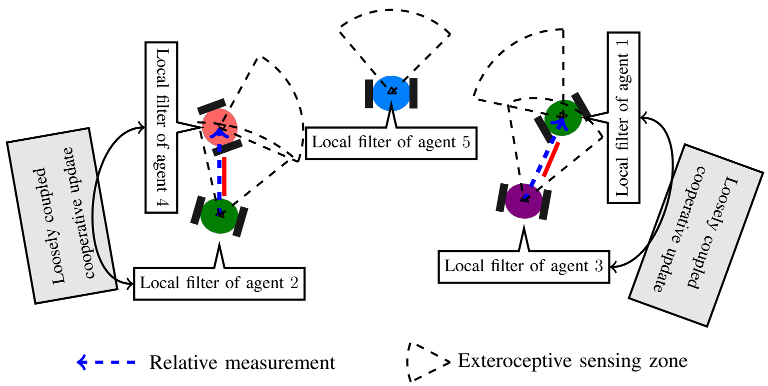


Fig. 1 – Schematic representation of our loosely coupled cooperative localization algorithm.

their exteroceptive sensors' measurement range. Therefore, if there is a relative measurement between to agents, it is safe to assume that they can communicate with each other.

Under the described setting, let each agent $i \in \{1, \cdots, N\}$ start with an initial estimate $\hat{\mathbf{x}}^{i+}(0) \in \mathbb{R}^{n^i}$ and $\mathbf{P}^{i+}(0) \in \mathbb{R}^{n^i \times n^i}$. Relying on the proprioceptive sensors, each agent i measures its ego-motion, and propagates its own local estimate according to the motion model (1) by

$$\hat{\mathbf{x}}^{i-}(k+1) = \mathbf{F}^i(k)\hat{\mathbf{x}}^{i+}(k) + \mathbf{B}^i(k)\mathbf{u}^i(k), \tag{4}$$

and the corresponding to covariance matrix of robot i will be given by

$$\mathbf{P}^{i-}(k+1) = \mathbf{F}^i(k)\mathbf{P}^{i+}(k)\mathbf{F}^i(k)^\top + \mathbf{G}^i(k)\mathbf{Q}^i(k)\mathbf{G}^i(k)^\top. \tag{5}$$

If mobile agents only rely on proprioceptive sensors to estimate their own states, the estimate error will drift which leads to the unbounded accumulation of error and uncertainty, which can be seen clearly from (5). To bound the error and uncertainty achieving better estimates, exteroceptive measurements are necessarily taken into account to update the estimates. Both absolute measurements and relative measurements can be used to update the estimates. When there is an occasional absolute measurement, modeled as in (3), agent i updates its local estimate using regular Kalman update equations.

Next, we describe our consistent relative measurement processing procedure. Without loss of generality assume that at some time k agent i takes relative measurement $z_{ij}(k)$ from agent j. Let $(\hat{\mathbf{x}}^{l-}(k),\, \mathbf{P}^{l-}(k)))$, $l \in \{i, j\}$, be the local state estimate and its associated error covariance of agent l prior to updating them using relative measurement between them. The idea here is to consider the states of agents i and j as joint state $\mathbf{x}_J$, i.e.,

$$\mathbf{x}_J(k) = \begin{bmatrix} \mathbf{x}^j(k) \\ \mathbf{x}^j(k) \end{bmatrix},$$

with aggregated propagated/updated states prior to relative measurement processing as

$$\hat{\mathbf{x}}_J^{\text{-}}(k) = \begin{bmatrix} \hat{\mathbf{x}}^{j\text{-}}(k) \\ \hat{\mathbf{x}}^{j\text{-}}(k) \end{bmatrix}.$$

At time k, prior to the relative measurement update, the joint covariance matrix is $\mathbf{P}_J^{\text{-}}$(k)

$$\mathbf{P}_J^{\text{-}} = \begin{bmatrix} \mathbf{P}^{i\text{-}}(k) & \mathbf{P}_{ij}^{\text{-}}(k) \\ \mathbf{P}_{ij}^{\text{-}\top}(k) & \mathbf{P}^{j\text{-}}(k) \end{bmatrix}. \tag{6}$$

where the cross-covariance $\mathbf{P}_{ij}^{\text{-}}(k)$ is unknown. Our objective is to update $\mathbf{x}_J^{\text{-}}(k)$ in a consistent manner using the relative measurement (2). To account for the unknown cross-covariance matrix, we invoke Lemma 2.1 to over-estimate the joint covariance by

$$\bar{\mathbf{P}}_J^{\text{-}}(k) = \begin{bmatrix} \frac{1}{\omega}\mathbf{P}^{i\text{-}}(k) & \mathbf{0} \\ \mathbf{0} & \frac{1}{1-\omega}\mathbf{P}^{j\text{-}}(k) \end{bmatrix} \geq \mathbf{P}_J^{\text{-}}(k), \quad \forall\, \omega \in [0,1]. \tag{7}$$

That is, in the update process the joint aggregated estimate is assume to be $(\hat{\mathbf{x}}_J^{\text{-}}(k), \bar{\mathbf{P}}_J^{\text{-}}(k))$. The states are updated according to

$$\mathbf{x}_J^{+}(k) = \mathbf{x}_J^{\text{-}}(k) + \mathbf{K}\,\mathbf{r}_{ij}(k), \tag{8}$$

where

$$\mathbf{r}_{ij}(k) = \mathbf{z}_{ij}(k) - \mathbf{H}_{ij}^i(k)\mathbf{x}^{i\text{-}}(k) - \mathbf{H}_{ij}^j(k)\mathbf{x}^{j\text{-}}(k).$$

For an optimal update we can use

$$\mathbf{K} = \operatorname{argmin} \operatorname{Tr}(E[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top]),$$

where $\tilde{\mathbf{x}}_J(k) = (\mathbf{x}_J(k) - \hat{\mathbf{x}}_J^{+}(k))$. However, because in the joint covariance matrix, the cross-covariance term between the estimates of agents i and j is unknown, we use the over-estimated joint covariance matrix $\bar{\mathbf{P}}_J$ to obtain the Kalman gain from

$$\mathbf{K} = \operatorname{argmin} \operatorname{Tr}\big((\mathbf{I} - \mathbf{K}\mathbf{H})\bar{\mathbf{P}}_J^{\text{-}}(\mathbf{I} - \mathbf{K}\mathbf{H})^\top + \mathbf{K}\mathbf{R}_r^i\mathbf{K}^\top\big),$$

which obtains $\mathbf{K}$ such that an upper bound on $\text{Tr}(E[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top])$ is minimized. Proceeding with manipulation similar those used in the derivation of Kalman filter equations, we obtain the Kalman gain and the corresponding covariance update as follows

$$\begin{aligned}\mathbf{K} = \begin{bmatrix}\mathbf{K}_i \\ \mathbf{K}_j\end{bmatrix} &= \bar{\mathbf{P}}_J^{-}\mathbf{H}^\top\mathbf{S}_{ij}^{-1} \\ &= \begin{bmatrix}\frac{1}{\omega}\mathbf{P}^{i-}(k)\mathbf{H}_{ij}^{i}{}^\top\mathbf{S}_{ij}^{-1} \\ \frac{1}{1-\omega}\mathbf{P}^{j-}(k)\mathbf{H}_{ij}^{j}{}^\top\mathbf{S}_{ij}^{-1}\end{bmatrix},\end{aligned} \tag{9}$$

and

$$\begin{aligned}\mathbf{P}_J^{+} &= (\mathbf{I}-\mathbf{K}\mathbf{H})\bar{\mathbf{P}}_J^{-}(\mathbf{I}-\mathbf{K}\mathbf{H})^\top + \mathbf{K}\mathbf{R}_r^i\mathbf{K}^\top \\ &= \bar{\mathbf{P}}_J^{-} - \mathbf{K}\,\mathbf{S}_{ij}\,\mathbf{K}^\top,\end{aligned} \tag{10}$$

where

$$\begin{aligned}\mathbf{S}_{ij} &= \mathbf{H}\bar{\mathbf{P}}_J^{-}\mathbf{H}^\top + \mathbf{R}_r^i(k) \\ &= \frac{1}{\omega}\,\mathbf{H}_{ij}^{i}\mathbf{P}^{i-}(k)\mathbf{H}_{ij}^{i}{}^\top + \frac{1}{1-\omega}\mathbf{H}_{ij}^{j}\mathbf{P}^{j-}(k)\mathbf{H}_{ij}^{j}{}^\top + \mathbf{R}_r^i(k).\end{aligned} \tag{11}$$

Here, we used

$$\mathbf{H} = \begin{bmatrix}\mathbf{H}_{ij}^{i}(k) & \mathbf{H}_{ij}^{j}(k)\end{bmatrix}. \tag{12}$$

Next, we show that this over-estimation of the joint covariance matrix, results in consistent updated estimates for agent i and j using relative measurements between them.

Theorem 3.1 (Consistency of the joint state estimate update using Kalman gain (9)*): Let agent i and agent j have unbiased and consistent estimates ($\hat{\mathbf{x}}^{i-}(k)$,$\mathbf{P}^{i-}(k)$) and ($\hat{\mathbf{x}}^{j-}(k)$,$\mathbf{P}^{j-}(k)$), respectively, at time k. The updated joint estimate $(\hat{\mathbf{x}}_J^{+}(k), \mathbf{P}_J^{+}(k))$ where $\hat{\mathbf{x}}_J^{+}$ and $\mathbf{P}_J^{+}$ are obtained from* (8) *and* (10) *with Kalman gain given in* (9) *is an unbiased and consistent estimate.*

Proof- Given that the propagated joint estimate is unbiased ($\text{E}[\mathbf{x}_J(k) - \hat{\mathbf{x}}_J^{-}(k)] = \mathbf{0}$.), and the measurement noise is zero mean, the joint updated state is unbiased, i.e., $\text{E}[\mathbf{x}_J(k) - \hat{\mathbf{x}}_J^{+}(k)] = \mathbf{0}$.

To prove consistency in the sense we defined in Definition 1, we need to show that

$$\mathbf{P}_J^{+} \geq \text{E}[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top],$$

where $\tilde{\mathbf{x}}_J(k) = (\mathbf{x}_J(k) - \hat{\mathbf{x}}_J^{+}(k))$. To this end, recall that

$$\mathbf{P}_J^{+} = (\mathbf{I} - \mathbf{K}\mathbf{H})\bar{\mathbf{P}}_J^{-}(\mathbf{I} - \mathbf{K}\mathbf{H})^\top + \mathbf{K}\mathbf{R}_r^i\mathbf{K}^\top$$

and

$$\mathrm{E}[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top] = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_J^{-}(\mathbf{I} - \mathbf{K}\mathbf{H})^\top + \mathbf{K}\mathbf{R}_r^i\mathbf{K}^\top,$$

where $\mathbf{P}_J^{-}$ is the joint covariance matrix between estimates of agent i and j as described in (6) with unknown cross-covariance matrix. Then,

$$\mathbf{P}_J^{+} - \mathrm{E}[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top] = (\mathbf{I} - \mathbf{K}\mathbf{H})(\bar{\mathbf{P}}_J^{-} - \mathbf{P}_J^{-})(\mathbf{I} - \mathbf{K}\mathbf{H})^\top,$$

which given (7), guarantees $\mathbf{P}_J^{+} - \mathrm{E}[\tilde{\mathbf{x}}_J(k)\tilde{\mathbf{x}}_J(k)^\top] \geq \mathbf{0}$, and thus, the consistency of the updated estimate. □

Theorem 3.1 showed that for any $\omega \in [0, 1]$ updating the state estimates of agents i and j using the gain (9) results in a consistent estimate. Next, to achieve the maximum benefit in reducing the joint uncertainty we obtain an optimal $\omega^\star \in [0, 1]$ from

$$\omega^\star = \underset{\omega \in [0,1]}{\operatorname{argmax}} \det(\mathbf{P}_J^{+\,-1}), \tag{13}$$

which can be cast in an equivalent convex optimization problem form

$$\begin{aligned}\omega^\star &= \underset{\omega \in [0,1]}{\operatorname{argmin}} \; -\log\det(\mathbf{P}_J^{+\,-1}) \\ &= \underset{\omega \in [0,1]}{\operatorname{argmin}} \; -\log\det(\bar{\mathbf{P}}_J^{-\,-1} + \mathbf{H}^\top\mathbf{R}_r^{i\,-1}\mathbf{H}),\end{aligned} \tag{14}$$

that is

$$\omega^\star = \underset{\omega \in [0,1]}{\operatorname{argmin}} \; -\log\det(\begin{bmatrix}\omega\mathbf{P}^{i-\,-1} & \mathbf{0} \\ \mathbf{0} & (1-\omega)\mathbf{P}^{j-\,-1}\end{bmatrix} + \mathbf{H}^\top\mathbf{R}_r^{i\,-1}\mathbf{H}).$$

Here, we used

$$\begin{aligned}\mathbf{P}_J^{+} &= \bar{\mathbf{P}}_J^{-} - \mathbf{K}\mathbf{S}_{ij}^{-1}\mathbf{K}^\top \\ &= \bar{\mathbf{P}}_J^{-} - \bar{\mathbf{P}}_J^{-}\mathbf{H}^\top(\mathbf{H}\bar{\mathbf{P}}_J^{-}\mathbf{H}^\top + \mathbf{R}_r^i)^{-1}\mathbf{H}\bar{\mathbf{P}}_J^{-},\end{aligned}$$

and matrix inversion Lemma to write

$$\mathbf{P}_J^{+\,-1} = \bar{\mathbf{P}}_J^{-\,-1} + \mathbf{H}^\top\mathbf{R}_r^{i\,-1}\mathbf{H}.$$

We used $\det(\mathbf{P}_J^{+})$ as our measure of total uncertainty. Alternatively, we can use $\mathrm{Tr}(\mathbf{P}_J^{+})$ as the measure of total uncertainty and obtain $\omega^{\star}$ from minimizing the trace uncertainty measure.

Remark 3.1 (Uncertainty reduction in the loosely coupled cooperative localization algorithm): The prior covariance matrix of each robot is $\mathbf{P}^{i\text{-}}$ and $\mathbf{P}^{j\text{-}}$. The joint covariance after the update is given by (10) whose elements for each agent read as follows (recall $\mathbf{S}_{ij}$ in (11))

$$\begin{aligned}\mathbf{P}^{i+} =&\frac{1}{\omega^{\star}}\mathbf{P}^{i\text{-}} - \mathbf{K}_i\mathbf{S}_{ij}\mathbf{K}_i^{\top} &\text{(15a)}\\ =&\frac{1}{\omega^{\star}}\mathbf{P}^{i\text{-}} - (\frac{1}{\omega^{\star}})^2\mathbf{P}^{i\text{-}}\mathbf{H}_{ij}^{i\,\top}\mathbf{S}_{ij}^{-1}\mathbf{H}_{ij}^{i}\mathbf{P}^{i\text{-}},\\ \mathbf{P}^{j+} =&\frac{1}{1-\omega^{\star}}\mathbf{P}^{j\text{-}} - \mathbf{K}_j\mathbf{S}_{ij}\mathbf{K}_j^{\top} &\text{(15b)}\\ =&\frac{1}{1-\omega^{\star}}\mathbf{P}^{j\text{-}} - (\frac{1}{1-\omega^{\star}})^2\mathbf{P}^{j\text{-}}\mathbf{H}_{ij}^{j\,\top}\mathbf{S}_{ij}^{-1}\mathbf{H}_{ij}^{j}\mathbf{P}^{j\text{-}},\end{aligned}$$

where recall that $\omega^{\star} \in [0,1]$ and as a result we have $\frac{1}{\omega^{\star}} \geq 0$ and $\frac{1}{1-\omega^{\star}} \geq 1$. In the update procedure to preserve filter consistency in the presence of unknown prior cross covariance, we first over estimate the covariance. Then the uncertainty is reduced from this enlarged uncertainty by using the relative measurement processing as we can see in (15). Using the determinant of the local updated covariance as our measure of uncertainty of each agent, we can use $g^{a}(\omega^{\star})$, $a \in \{i,j\}$, defined as follows, as a measure to devalue the reduction in the uncertainty of the agents (recall (15))

$$\det(\mathbf{P}^{i+}) = g^{i}(\omega)\det(\mathbf{P}^{i\text{-}}), \quad \det(\mathbf{P}^{j+}) = g^{j}(\omega)\det(\mathbf{P}^{j\text{-}}), \tag{16}$$

where

$$\begin{aligned}g^{i}(\omega) =&\det\Big(\frac{1}{\omega}\mathbf{I} - (\frac{1}{\omega})^2\sqrt{\mathbf{P}^{i\text{-}}}^{\top}\mathbf{H}_{ij}^{i\,\top}\mathbf{S}_{ij}^{-1}\mathbf{H}_{ij}^{i}\sqrt{\mathbf{P}^{i\text{-}}}\Big),\\ g^{j}(\omega) =&\det\Big(\frac{1}{1-\omega}\mathbf{I} - (\frac{1}{1-\omega})^2\sqrt{\mathbf{P}^{j\text{-}}}^{\top}\mathbf{H}_{ij}^{j\,\top}\mathbf{S}_{ij}^{-1}\mathbf{H}_{ij}^{j}\sqrt{\mathbf{P}^{j\text{-}}}\Big).\end{aligned}$$

□

The experiment depicted in Fig. IV demonstrates the reduction in the uncertainty of the mobile agents after implementing our proposed loosely coupled algorithm for cooperative localization (the blue circles highlight the onset of implementing our propose algorithms)–for details of this numerical experiment please see Section IV.

In the following, we discuss an alternative approach to design the optimal ω in a manner that we can place weight on which agent to receive more improvement from the cooperative localization update. We use, once again, the determinant of the local updated covariance as our measure of uncertainty of each agent. Remark that applying the matrix inversion Lemma on (15), we can obtain the inverse of the local updated covariance matrices as follows

$$\mathbf{P}^{i+^{-1}} = \omega\,\mathbf{P}^{i-^{-1}} + (1-\omega)\,{\mathbf{H}_{ij}^{i}}^{\top}(\mathbf{H}_{ij}^{j}\mathbf{P}^{j-}{\mathbf{H}_{ij}^{j}}^{\top} + (1-\omega)\,\mathbf{R}_{r}^{i})^{-1}\mathbf{H}_{ij}^{i}, \tag{17a}$$

$$\mathbf{P}^{j+^{-1}} = (1-\omega)\,\mathbf{P}^{j-^{-1}} + \omega\,{\mathbf{H}_{ij}^{j}}^{\top}(\mathbf{H}_{ij}^{i}\mathbf{P}^{i-}{\mathbf{H}_{ij}^{i}}^{\top} + \omega\,\mathbf{R}_{r}^{i})^{-1}\mathbf{H}_{ij}^{j}. \tag{17b}$$

We define our alternative weighted objective function to obtain $\omega^{\star}$ as

$$\omega^{\star} = \underset{\omega\in[0,1]}{\operatorname{argmax}}\, c^{i}\log\det(\mathbf{P}^{i+^{-1}}) + c^{j}\log\det(\mathbf{P}^{j+^{-1}}). \tag{18}$$

Here, $c^i \geq 0$ and $c^j \geq 0$ are the weights which are introduced to adjust the gain each agent can achieve from the cooperative localization update. An interesting scenario that can be constructed from this optimization routine is when we allow one of the agents, say without loss of generality agent i, to act selfishly. That is, we let $c^j = 0$. This scenario would be of interest in cases that one of the team members is likely to have more accurate positioning estimates than the other teammate. For example in underwater operations around a surface mother-ship, the mother-ship because of its access to GPS, will have more accurate positioning than the underwater vehicles.

Remark 3.2 (Extension to local filters other than Kalman filter): In our algorithm, the cooperative relative measurement processing problem with unknown cross-covariance is changed into the joint update of $(\hat{\mathbf{x}}^{i-}, \frac{1}{w}\mathbf{P}^{i-})$ and $(\hat{\mathbf{x}}^{j-}, \frac{1}{1-w}\mathbf{P}^{j-})$ using the relative measurement, with no correlation between these estimates. Therefore, the local filters and the filter used to update the estimates is not restricted to be a Kalman filter and other estimation filters such as UKF can also be used to process the relative measurement. □

C. **Consistent loosely coupled cooperative navigation algorithms**

Using the theoretical developments in Section III-B, we propose two consistent loosely coupled cooperative navigation algorithms as described in Algorithm 1 and Algorithm 2. In Algorithm 1, which we refer to it as 'Mutualistic[1] Cooperative Localization', agents engaged in a relative

[1]In biology muturalism is a symbiotic relation that both organisms benefit from their cooperation.

measurement minimize their joint total uncertainty by implementing $\omega^\star$ obtained from (14). Moreover, both agents update their estimates, in a consistent manner, using the relative measurement between them. In Algorithm 2, which we refer to it as 'Commensalistic[2] Cooperative Localization', to obtain the update gain, one of the agents selfishly minimizing its own local updated covariance matrix with respect to ω (i.e., the selfish agent sets the weight of the other agent in (18) equal to zero). In our algorithms presentation here, for brevity, we only discuss the case where at each time each agent is only engaged in one relative measurement. We also do not discuss the details of processing absolute measurements. These case can be processed using sequential processing procedure (consult, for example, [31, Ch. 3], [32]). Details are omitted for brevity.

For any agent $a \in \{1, \cdots, N\}$, we let $\mathcal{V}^a_{\text{coop}}(k) \subseteq \{1, \cdots, N\}$ be the set of agents that agent $a \in \{1, \cdots, N\}$ prior to the current time k has updated its estimates by cooperating with them plus the agent a itself. Two observation are in order here. One is that the state estimate of agent a at time k is correlated with the state estimate of agents in $\mathcal{V}^a_{\text{coop}}(k)$. Also, at any time k, for any agent $b \in \mathcal{V}^a_{\text{coop}}(k)$, the state estimate of agent a is correlated with the state estimate of all the agents in $\mathcal{V}^b_{\text{coop}}(k)$. In our algorithms described below, we do not keep an accurate account of these correlations but account for them in an implicit manner using the method we developed in Section III-B.

IV. **NUMERICAL SIMULATIONS**

We test the performance of our proposed loosely coupled consistent cooperative localization algorithms (Mutualistic Cooperative Localization and Coomensalistic Cooperative Localization) in simulations for a group of 3 robots moving on a flat terrain. The simulations are run for 1000 time steps (each time step has a duration of 0.1 sec). Each robot measures its linear, ν, and angular velocity, ω, with independent zero-mean white Gaussian noises with standard deviation $\sigma_\nu = 0.1\, m/s$ and $\sigma_\omega = 2\, deg/s$. The relative measurements used in the simulation are relative range and relative bearing. We set the measurement noise to be independent zero-mean white Gaussian distributed with standard deviation $\sigma_r = 0.05\, m$ and $\sigma_\phi = 1\, deg$. Each robot propagates its own state at every time step while the measurement update happens when relative

[2]In biology commensalism is a symbiotic relation that one of the organisms benefit and the other is unaffected

Algorithm 1 Loosely Coupled Mutualistic Cooperative Localization

Require: Initialization (k=0):

every agent $i \in \{1, \cdots, N\}$ initializes its estimate at $\hat{\mathbf{x}}^{i+}(0) \in \mathbb{R}^{n^i}$ and $\mathbf{P}^{i+}(0) \in \mathbb{S}_{n^i}^{++}$

Iteration k+1:

Propagation: Every agent i propagates its own state locally according to (4) and (5).

Update:

- If there is no relative measurement involving agent i, agent i sets its updated estimate to its propagated one.
- If agent i takes a relative measurement from agent j, then agent i sends $\big(\mathbf{z}_{ij}(k+1), \mathbf{R}_r^i(k+1), \hat{\mathbf{x}}^{i-}(k+1), \mathbf{P}^{i-}(k+1), \mathcal{V}^i(k+1)\big)$ to agent j. Next, agent j checks if there is a correlation between its state estimate and that of i by checking whether $\{\mathcal{V}^j(k+1) \cap \mathcal{V}^j(k+1) = \{\}$ or not.
  - If the answer to this prob is yes, then there is no correlation between state estimate of agent i and that of j. Therefore, agent j proceeds with updating their joint estimates using (8) and $\mathbf{P}_J^{+}(k+1) = \mathbf{P}_J^{-}(k+1) - \mathbf{K}\mathbf{S}_{ij}^{-1}\mathbf{K}^\top$, where
$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_i \\ \mathbf{K}_j \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{i-}(k+1)\mathbf{H}_{ij}^{i\,\top}\mathbf{S}_{ij}^{-1} \\ \mathbf{P}^{j-}(k+1)\mathbf{H}_{ij}^{j\,\top}\mathbf{S}_{ij}^{-1} \end{bmatrix}, \tag{19}$$
    and $\mathbf{S}_{ij} = \mathbf{H}_{ij}^i\mathbf{P}^{i-}(k+1)\mathbf{H}_{ij}^{i\,\top} + \mathbf{H}_{ij}^j\mathbf{P}^{j-}(k+1)\mathbf{H}_{ij}^{j\,\top} + \mathbf{R}_r^i(k+1)$.
  - If the answer to this prob is no, then there is an unknown correlation between estimates of agent i and j. Then agent j uses (8) and (10), where $\omega = \omega^\star$ is computed from (13), to update its own estimate and that of agent i.

  Agent j then uses $\big(\hat{\mathbf{x}}^{j+}(k+1), \mathbf{P}^{j+}(k+1)\big)$ as its current updated estimate and sends back $\big(\hat{\mathbf{x}}^{i+}(k+1), \mathbf{P}^{i+}(k+1)\big)$ to agent i as that agent's current updated estimate.

$k \leftarrow k+1$

End for

Algorithm 2 Loosely Coupled Commensalistic Cooperative Localization

Require: Initialization (k=0):

every agent $i \in \{1, \cdots, N\}$ initializes its estimate at $\hat{\mathbf{x}}^{i+}(0) \in \mathbb{R}^{n^i}$ and $\mathbf{P}^{i+}(0) \in \mathbb{S}_{n^i}^{++}$

Iteration k+1:

Propagation: Every agent i propagates its own state locally according to (4) and (5).

Update:

- If there is no relative measurement involving agent i, agent i sets its updated estimate to its propagated one.
- If agent i takes a relative measurement from agent j, and without loss of generality agent j is the selfish agent, then agent i sends $\big(\mathbf{z}_{ij}(k+1), \mathbf{R}_r^i(k+1), \hat{\mathbf{x}}^{i-}(k+1), \mathbf{P}^{i-}(k+1), \mathcal{V}^i(k+1)\big)$ to agent j. Next, agent j checks if there is a correlation between its state estimate and that of i by checking whether $\{\mathcal{V}^j(k+1) \cap \mathcal{V}^j(k+1) = \{\}$ or not.
  - If the answer to this prob is yes, then there is no correlation between state estimate of agent i and that of j. Therefore, agent j proceeds with updating their joint estimates using (8) and $\mathbf{P}_J^{+}(k+1) = \mathbf{P}_J^{-}(k+1) - \mathbf{K}\mathbf{S}_{ij}^{-1}\mathbf{K}^\top$, where the update gain is computed from (19) with $\mathbf{S}_{ij} = \mathbf{H}_{ij}^i \mathbf{P}^{i-}(k+1) \mathbf{H}_{ij}^{i\,\top} + \mathbf{H}_{ij}^j \mathbf{P}^{j-}(k+1) \mathbf{H}_{ij}^{j\,\top} + \mathbf{R}_r^i(k+1)$.
  - If the answer to this prob is no, then there is an unknown correlation between estimates of agent i and j. Then agent j uses (8) and (10), where $\omega = \omega^\star$ is computed from (18) with $c^i = 0$ and $c^j = 1$, to update the state estimates.

  Then, agent j uses $\big(\hat{\mathbf{x}}^{j+}(k+1), \mathbf{P}^{j+}(k+1)\big)$ as its current estimate and sends back $\big(\hat{\mathbf{x}}^{i+}(k+1), \mathbf{P}^{i+}(k+1)\big)$ to agent i as that agent's current updated estimate.

$k \leftarrow k+1$

End for

measurement is detected.

Our first simulation study depicted in Fig. IV compares the performance of our proposed Mutualistic Cooperative Localization algorithm to that of pure propagation and that of cooperative localization with no regards to past correlations. The estimation error (solid lines) and the 3σ error bounds (dashed lines) are as shown in Fig. IV. Here, $a \to b$ over the time interval marked by two vertical black lines indicates that robot a has taken a relative measurement with respect to robot b at that time interval. The symbol $a \to a$ means that robot a obtains an absolute measurement.

At the time step $t = 20s$, agent 1 take relative measurement from agent 2 for the first time. As there is no correlation yet between agent 1 and agent 2, independence holds here at this instant. Afterwards, the correlation between agent 1 and 2 is established and it should be taken into account. At the time step $t = 40s$, similarly, agent 2 takes its first relative measurement from agent 3. Once again as there is no correlation between these two robots' estimates, independence holds. Afterwards, the correlation should be taken into account. As Fig. IV shows, ignoring the past correlations leads to inconsistent estimates (see red plots). Recall that the correlation is not only established by direct relative measurement update between two robots but also by indirect manner as well. For example, at time $t = 70s$, agent 3 takes a relative measurement from agent 1 for the first time. But at this point, their estimates are correlated because of both simultaneously being correlated with robot 2. As shown, the proposed method without keeping track of the cross-covariances is consistent and has a significant reduction of uncertainty compared to the pure propagation case. As we can see from $t = 60s$ to $t = 70s$, as agent 1 takes benefit from the absolute measurements, it pose accuracy improves significantly. When agent 3 takes relative measurement from agent 1 afterwards, the high accuracy of agent 1 also benefits agent 3. Next, the root mean square error (RMSE) criterion was employed to test the accuracy (averaged over 50 Monte Carlo runs) as shown in Fig. 3. A reduction of RMSE can be observed for the proposed method.

Our second simulation study depicted in Fig. 4 compares the performance of our proposed Mutualistic Cooperative Localization algorithm (thin red plots) and the Commensalistic Cooperative Localization (thick black plots) for two robots. Here, we assume that robot 1 needs higher accuracy in its positioning therefore in the Commensalistic Cooperative Localization algorithm we let this agent to be the selfish agent, i.e., at the time of relative measurement processing we set $c^1 = 1$ and $c^2 = 0$ in (18). As Fig. 4 shows in the Commensalistic Cooperative Localization algorithm, agent 1 acquires higher precision than in Mutualistic Cooperative Localization algorithm.

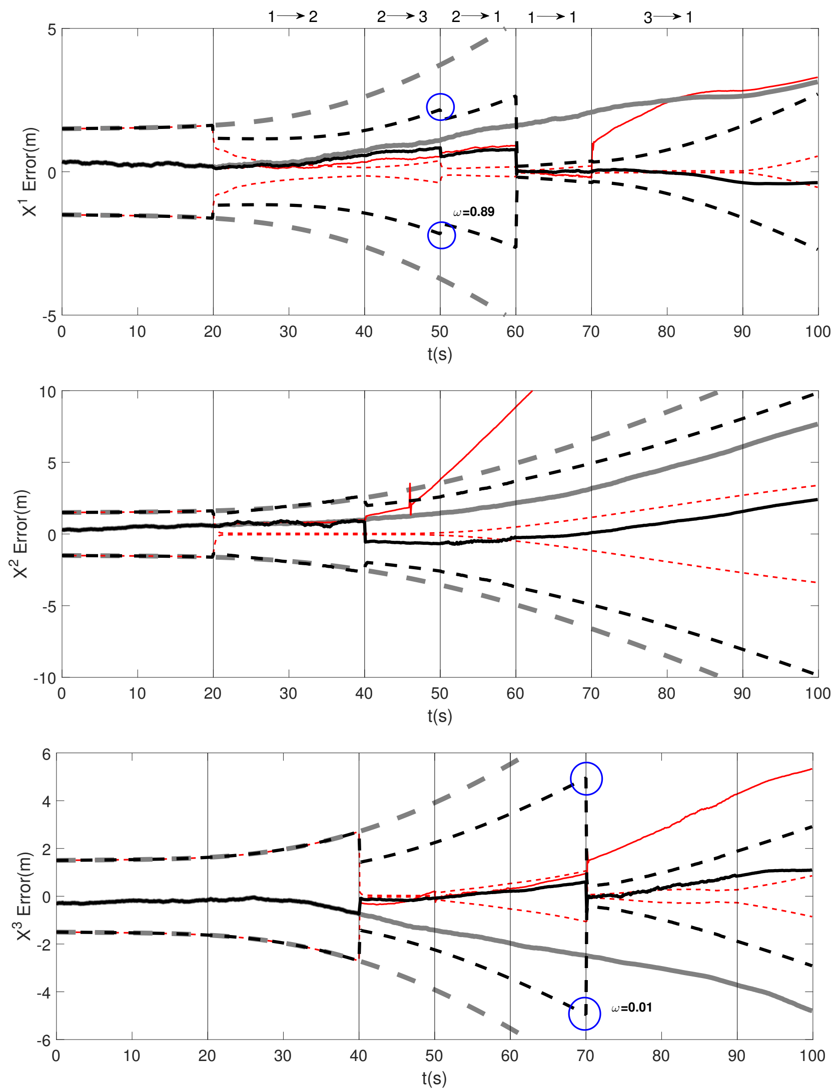


Fig. 2 – Estimation error (solid line) and the 3σ error bounds (dashed line) for three agents moving on a flat terrain when they (a) only propagate their equation of motion using self-motion measurements (thick grey plots), (b) employ cooperative localization ignoring past correlations between the estimates of the agents (red plots), and (c) employ the proposed consistent loosely coupled decentralized cooperative navigation Algorithm 1 (black plots). As these plots show, cooperative localization improves the positioning accuracy of the robots. The plots also show that ignoring the past correlations can result in overly optimistic and inconsistent estimates.

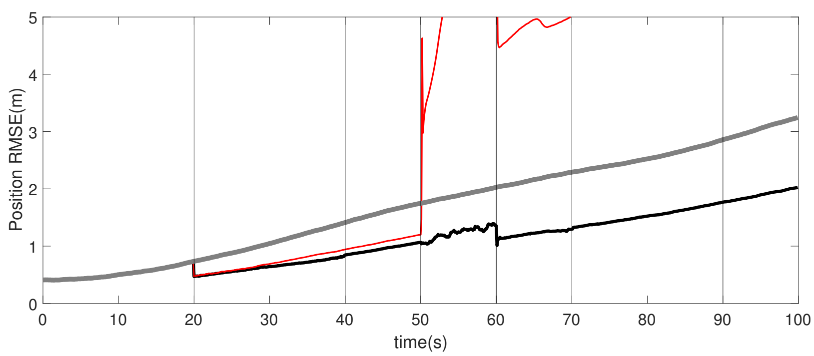
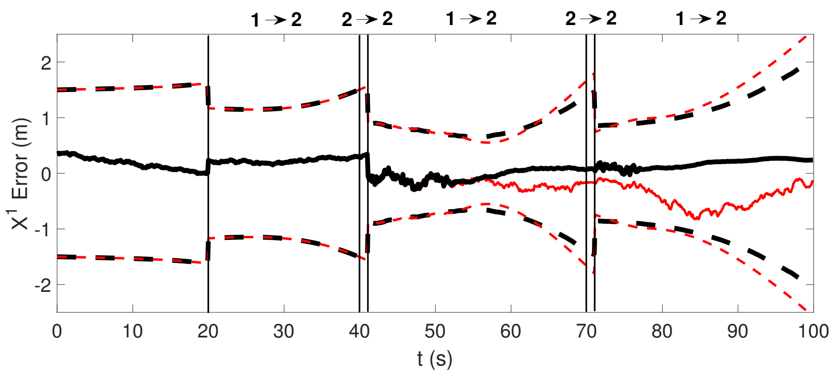
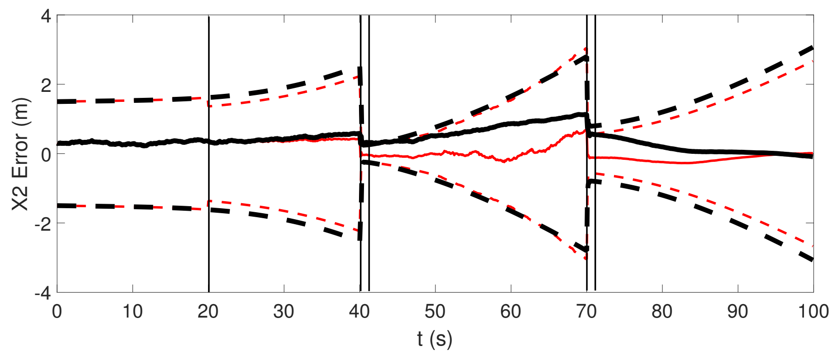


Fig. 3 – The root mean square error (RMSE) of the agents for the simulation averaged over 50 Monte Carlo runs and over all agents. The agents (a) only propagate their equation of motion using self-motion measurements (thick grey plots), (b) employ cooperative localization ignoring past correlations between the estimates of the agents (red plots), and (c) employ the proposed consistent loosely coupled decentralized cooperative navigation Algorithm 1 (black plots). As seen, cooperative localization via Algorithm 1 results in improving the position accuracy of the robots which cooperative localization ignoring past correlations results in eventually highly inaccurate positioning results, to the point that is is not seen in the window shown.



Agent 1



Agent 2

Fig. 4 – Estimation error (solid line) and the 3σ error bounds (dashed line) for two agents moving on a flat terrain when they (a) employ the Mutualistic Cooperative Localization (thin red plots), and (c) employ the Commensalistic Cooperative Localization (thick black plots). As these plots show, employing the Commensalistic cooperative localization results in higher accuracy for robot 1 which is assumed to be the selfish agent.

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